

Set Theory and Proofs for Engineering Education

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Abstract

Through evaluations of learning objectives on several Engineering courses, the majority of students at some point will struggle with demonstration of proof of a principle in their homework assignments, quizzes, and exams. An early introduction of "Set Theory and Proofs" to engineering students can enrich their intuition and ability to solve comprehensive problems. As illustrated in this paper, set theory can be recognized by students as a simple and unnecessary topic. However, the understanding of principles in set theory and its derived concepts are essential to engineering students so they can improve their problem-solving skills when approaching a more complex problem using mathematics. Set Theory is a vast field of study which includes: Operations and algebra with sets, power sets, product sets, relations, functions, quantifiers, family of sets, index sets, just to name a few [1]. At The University of Texas at Tyler, the authors experienced set theory embedded in the learning objectives of Manufacturing Systems (MENG 5318) course offered by the Mechanical Engineering Department to its graduate students. In Fall 2016 and 2017, most of the students in the class failed to apply some of the principles in set theory. Overall, set theory is an important topic to engineering students where an understanding of the principal will ensure the success in completing advanced level courses.

1. Introduction

A set is any collection of define, distinguishable objects. These objects are called the elements or member of a set, e.g. $A = \{a, b, c, 2, 3\}$ and $B = \{1, 2, 3\}$. Union (\cup), intersection (\cap), and subtraction ($-$) are the basic operation concepts in set theory [2].
 $A \cup B = \{a, b, c, 1, 2, 3\}$ * AUB: the union of set A and set B
 $A \cap B = \{2, 3\}$ * A∩B: the intersection of set A and set B
 $A - B = \{a, b, c\}$ $B - A = \{1\}$ * A - B: set A minus set B
 b is element of A: $b \in A$
 1 is element of B: $1 \in B$
 1 is not element of A: $1 \notin A$
 a is not element of B: $a \notin B$

2. A conditional, a universal set (U), a subset (\subseteq), a complement (c) of a set, and an empty set (\emptyset)[3].

A conditional, e.g. If I don't water my plant, then my plant will die.

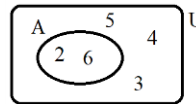
$M = \{a, b, c\}$ and $N = \{a, b, c\}$

We have that, $M = N$

If $M = N$, then $M \subseteq N$ and $N \subseteq M$ and

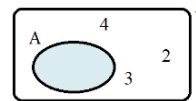
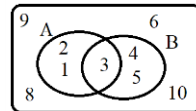
If $M \subseteq N$ and $N \subseteq M$, then $M = N$

Universal set: $U = \{2, 3, 4, 5, 6\}$ Set $A = \{2, 6\}$ Set A is subset of set U: $A \subseteq U$ The complement of set A: all the objects that do not belong to set A, $A^c = \{3, 4, 5\}$

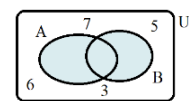


Complement of $(A \cup B)$: $(A \cup B)^c = \{6, 8, 9, 10\}$, the

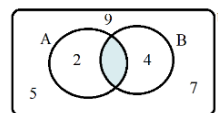
Complement of $(A \cap B)$: $(A \cap B)^c = \{1, 2, 4, 5, 6, 8, 9, 10\}$



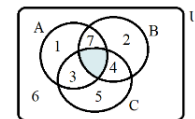
Set A is an empty set: $A = \emptyset$



$A \cup B = \emptyset$



$A \cap B = \emptyset$



$A \cap B \cap C = \emptyset$

3. Conditionals

3.1 Understanding conditionals, negation (\sim), the concepts “or (\vee)”, “and (\wedge)” and subsets in set theory [4].

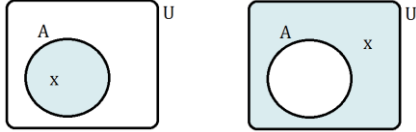
Let $x \in A \cap B$

If $x \in A \cap B$, then $x \in A$ and $x \in B$. See Fig. 1(a)

Let $x \in A \cup B$, implies that $x \in A$ or $x \in B$. See Fig. 1(b)

CASE 1: $x \in A$ If $x \in A$, then $x \notin B$. See Fig. 1(c)

CASE 2: $x \in B$ If $x \in B$, then $x \notin A$. See Fig. 1(d)



Let $x \in A$ If $x \in A$, then $x \notin A^c$

Let $x \in A^c$ If $x \in A^c$, then $x \notin A$. See figures above

Let $x \in A - B$

If $x \in A - B$, then $x \in A$, and $x \notin B$. See Fig. 2(a)

Let $x \in B - A$

If $x \in B - A$, then $x \in B$, and $x \notin A$. See Fig. 2(b)

Let $x \in A$

If $x \in A$, then $x \in A \cup B$. See Fig. 2(c)

Let $x \in B$

If $x \in B$, then $x \in (A \cup B)$. See Fig. 2(d)

Let $x \in (A \cap B)$

If $x \in (A \cap B)$, then $x \in A$, and $x \in B$. See Fig. 2(e)

Negation: \sim

$\sim(x \in A \cup B)$ *x is not element of $(A \cup B)$. See Fig. 4(a)

$x \in (A \cup B)^c$ *x is element of the complement of $A \cup B$. See Fig. 4(b)

$x \in (A \cup B)^c \Rightarrow x \notin (A \cup B) \Rightarrow \sim(x \in A \cup B)$ It reads, x is element of the complement of $A \cup B$ implies x is not element of $A \cup B$ implies the negation of $x \in (A \cup B)$.

$\sim(x \in A \cup B)$ implies $\sim(x \in A \vee x \in B)$ implies $(x \notin A \wedge x \notin B)$

*x is not element A and x is not element of B

Thus, $\sim(x \in A \cup B) \Rightarrow x \notin (A \cup B)$

If $x \notin (A \cup B)$, then $x \in (A \cup B)^c$

$\sim(x \in A \cap B)$ *x is not element of $(A \cap B)$

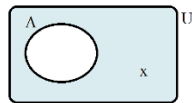
Implies, $\sim(x \in A \wedge x \in B)$ implies, $(x \notin A \vee x \notin B)$ *x is not element A or x is not element of B

$\sim(x \in A \cap B) \Rightarrow x \notin (A \cap B)$. See Fig. 4(c)

If $x \notin (A \cap B)$, then $x \in (A \cap B)^c$

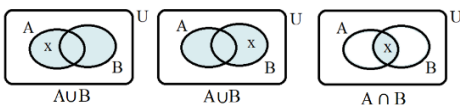
Knowing the concepts “or (\vee)”, “and (\wedge)” in set theory is important, and it can be very easy to get confuse with them when doing proofs.

x is not element A, implies $x \in A^c$,
implies x is element of the
complement of A: $\sim(x \in A) \Rightarrow x \in A^c$
 $\Rightarrow x \notin A$. If $x \notin A$, then $x \in A^c$



$B \subseteq A$ *B is subset of A. Let $x \in B$. If $x \in B$, then $x \in A$, see Fig. 3(a). Now, let $x \in A$. If $x \in A$ it does not imply that $x \in B$. See Fig. 3(b)

3.2 Definitions



$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$, it reads: $A \cup B = \{x \text{ is element of the universal set, such that } x \text{ is element of set } A \text{ or } x \text{ is element of set } B\}$. See $A \cup B$ figures above.

$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$, it reads $A \cap B = \{x \text{ is element of the universal set such that } x \text{ is element of } A \text{ and } x \text{ is element of } B\}$. See $A \cap B$ figure above.

Let $x \in (A \cup B \cup C)$ implies that $x \in A$ or $x \in B$ or $x \in C$. See Fig. 6(a)

Let $x \in A \cup (B \cap C)$ implies $x \in A$ or $x \in (B \cap C)$. See Fig.6(b)

Let $x \in (A \cup B) \cap (A \cup C)$ implies $x \in (A \cup B)$ or $x \in (A \cap C)$. See Fig.6(c)

4. Proofs[1]

*Prove that, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Proof:

Let $x \in A \cup (B \cap C)$, implies that $x \in A$ or $x \in B \cap C$

CASE 1: $x \in A$

If $x \in A$, then $x \in (A \cup B)$ and $x \in (A \cup C)$

Thus, $x \in (A \cup B) \cap (A \cup C)$

CASE 2: $x \in B \cap C$

If $x \in B \cap C$, then $x \in B$ and $x \in C$

Since $x \in B$ and $x \in C$, $x \in A \cup B$ and $x \in A \cup C$

Thus, $x \in (A \cup B) \cap (A \cup C)$

In either case $x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

*Prove that $A - B = A \cap B^c$ See fig. 5

Step 1. Prove that $A - B \subseteq A \cap B^c$

Proof:

Let $x \in A - B$

If $x \in A - B$, then $x \in A$ and $x \notin B$

If $x \notin B$, then $x \in B^c$

We have that $x \in A$ and $x \in B^c$

$\Rightarrow x \in A \cap B^c$

Thus, $A - B \subseteq A \cap B^c$

Step 2. Prove that $A \cap B^c \subseteq A - B$

Proof:

Let $x \in A \cap B^c$

If $x \in A \cap B^c$, then $x \in A$ and $x \in B^c$

If $x \in B^c$, then $x \notin B$

We have that $x \in A$ and $x \notin B$

$\Rightarrow x \in A - B$

Thus, $A \cap B^c \subseteq A - B$

Therefore, since $A - B \subseteq A \cap B^c$ and $A \cap B^c \subseteq A - B$,

$A - B = A \cap B^c$

5. Summary

Set theory is an interesting subject and an essential tool for doing proofs. It is encouraged that students need to be introduced to set theory early in their mathematical education. Set theory is a wide field of study, and its introduction to students should be started with the basic principles.

Acknowledgement

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References

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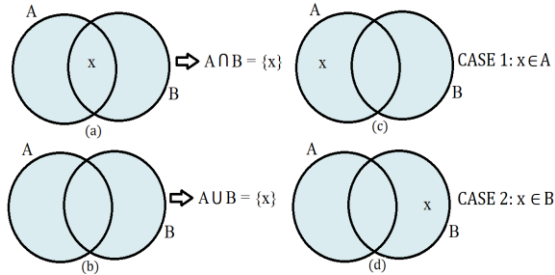


Fig. 1 (a), (b), (c), (d)

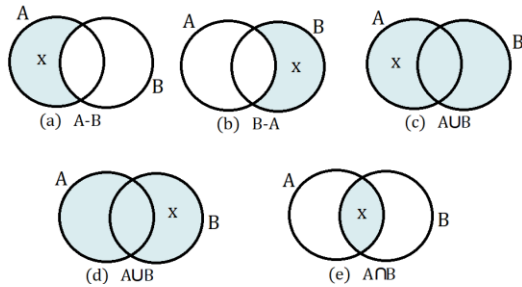


Fig. 2(a), (b), (c), (d)

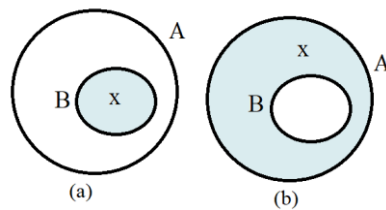


Fig. 3 (a), (b)

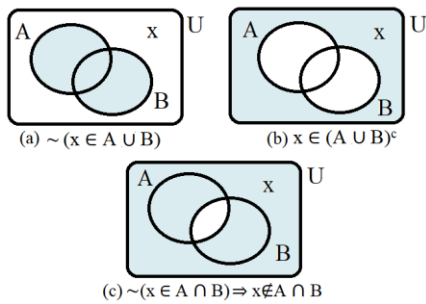


Fig. 4 (a), (b), (c)

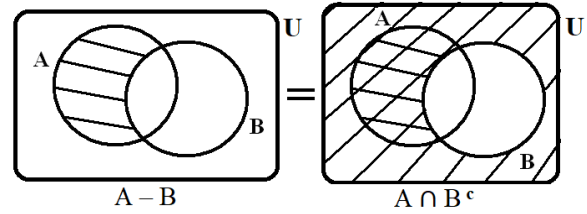


Fig. 5